Dirac's inspired point form and hadron form factors

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Noticing that the "point-form" approach referred to in many recent works implies physics described on hyperplanes, an approach inspired from Dirac's one, which involves a hyperboloid surface, is presented. A few features pertinent to this new approach are emphasized. Consequences as for the calculation of form factors are discussed.

1. INTRODUCTION

Looking at a Hamiltonian formulation of relativistic dynamics, Dirac was led to consider various forms, depending on the symmetry properties of the hypersurface that is chosen in this order [1]. Accordingly, the generators of the Poincaré algebra drop into dynamical or kinematic ones. Among the different forms, the point-form approach, which is based on a hyperboloid surface, $t^2 - \vec{x}^{\,2} = \tau$, is probably the most aesthetic one in the sense that the space-time displacement operators, P^μ, are the only ones to contain the interaction while the boost and rotation operators, $M^{\mu\nu}$ altogether, have a kinematic character. This approach is also the less known one, perhaps because dealing with a hyperboloid surface is not so easy as working with the hyperplanes that underly the other forms (instant and front). It nevertheless received some attention recently within the framework of relativistic quantum mechanics (RQM). Due to the kinematic character of boosts, its application to the calculation of form factors can be easily performed *a priori* and, moreover, these quantities generally evidence the important property of being Lorentz invariant.

A "point-form" ("P.F.") approach has been successfully used for the calculation of the nucleon form factors [2]. It however fails in reproducing the form factor of much simpler systems, including the pion [3, 4, 5]. The asymptotic behavior is missed and the drop-off at small Q^2 is too fast in the case of a strongly-bound system. Analyzing the results, it was found that this "point-form", where the dynamical or kinematic character of the Poincaré generators is the same as for Dirac's one, implies physics described on hyperplanes [6]. This approach is nothing but that one presented by Bakamjian as being an "instant form $\cdots$ which displays the symmetry properties inherently present in the point form" [7]. Sokolov mentioned it was involving "hyperplanes orthogonal $\cdots$ to the 4-velocity $\cdots$ of the system" under consideration, adding it was not identical to the point form proposed by Dirac [8]. Developing an approach more in the spirit of the original one therefore remains to be made.

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In this contribution, we present an exploratory work that is motivated by Dirac's point form and, consequently, implies physics described on hyperboloid-type surfaces. Due to the lack of space, we only consider here the main points while details can be found in Ref. [9]. How this new approach does for hadron form factors is briefly mentioned.

2. FORMALISM: A FEW INGREDIENTS

Each RQM approach is characterized by the relation that the momenta of a system and its constituents fulfill off-energy shell. This one is determined by the symmetry properties of the hypersurface which the physics is formulated on. In absence of particular direction on a hyperboloid-type surface, it necessarily takes the form of a Lorentz scalar. One should also recover the momentum conservation in the non-relativistic limit, $\sum_i \vec{p_i} = \vec{P}$. Thus, for the two-body system we are considering here, the expected relation could read:

$$(p_1 + p_2 - P)^2 = (\vec{p_1} + \vec{p_2} - \vec{P})^2 - (e_1 + e_2 - \; E_P)^2 = 0\,. \tag{1}$$

Such a constraint is obtained from integrating plane waves on the hypersurface, $x^2 = 0$:

$$\int d^4x \; e^{i(p-p')\cdot x} \; \delta(x^2) \; \epsilon(U\cdot x) = 4i\pi^2 \; \delta((p-p')^2) \; \epsilon(U\cdot(p-p'))\,, \tag{2}$$

where p^μ and p'^μ are replaced by $(p_1{+}p_2)^\mu$ and P^μ, and U^μ satisfies $U^2 \geq 0$. To understand the ingredients entering the l.h.s. of the above equation, the "time" evolution should be examined [9]. This goes beyond considering the upper part of a hyperboloid surface often mentioned in the literature. Interestingly, Eq. (1) can be cast into the following form:

$$(\vec{p_1} + \vec{p_2} - \vec{P}) = \hat{u} \; (e_1 + e_2 - \; E_P) \; (\hat{u}^2 = 1, \; \hat{u} \text{ not fixed})\,, \tag{3}$$

which is very similar to a front-form one, but the unit vector, $\hat{u}$, has no fixed direction.

The next step consists in considering a wave equation, which can be obtained from taking the square of the momentum operator, P^μ:

$$\begin{aligned}\Big(M^2 - p^2\Big) \; \Phi_P(\vec{p}_1, \vec{p}_2) = &-\int\!\!\int \frac{d\vec{p}^{\,\prime}_1}{(2\,\pi)^3} \; \frac{d\vec{p}^{\,\prime}_2}{(2\,\pi)^3} \; \frac{1}{(2\,e_1\;2\,e_2\;2\,e'_1\;2\,e'_2)^{1/2}} \\ &\times (p+p')\cdot\, \partial_{p-p'} \bigg(4\,\pi^2\;\delta\Big((p-p')^2\Big)\;\epsilon\Big(U\cdot(p-p')\Big)\bigg) \; \frac{4\,m^2\,g^2}{\mu^2 + \cdots} \;\; \Phi_P(\vec{p}^{\,\prime}_1, \vec{p}^{\,\prime}_2)\,,\end{aligned} \tag{4}$$

where $p^\mu = p_1^\mu + p_2^\mu$. One should determine under which conditions it admits solutions verifying Eq. (1) and, at the same time, leads to a relevant mass operator. With this aim, we assume $U^\mu \propto c(p-P)^\mu + c'(p'-P)^\mu$, from which we get:

$$\frac{\vec{U}}{U^0} = \hat{u} = \frac{\vec{p}-\vec{P}}{e-E_P} = \frac{\vec{p}^{\,\prime}-\vec{P}}{e'-E_P} = \frac{\vec{p}^{\,\prime\prime}-\vec{P}}{e''-E_P} = \cdots\,. \tag{5}$$

This relation shows that the orientation of $\hat{u}$ is conserved, which greatly facilitates the search for a solution. While doing so, a Lorentz-type transformation adapted from the Bakamjian-Thomas one [10] has to be made. The constituent momenta are expressed in terms of the total momentum, $\vec{P}$, and the internal variable, $\vec{k}$, while verifying Eq. (1). Moreover, the interaction is assumed to fulfill constraints but these ones, which amount to take into account higher-order meson-exchange contributions, are actually well known as part of the general construction of the Poincaré algebra in RQM approaches [11].

The present point form implies that the system described in this way evidences a new degree of freedom. In the c.m., a zero total momentum is obtained by adding the individual contributions of constituents and an interaction one, consistently with the fact that $\vec{P}$ contains the interaction. The configuration so obtained points isotropically to all directions as sketched in Fig. 2 of Ref. [9]. This new degree of freedom appears explicitly in the definition of the norm, beside the integration on the internal $\vec{k}$ variable:

$$N = \int \frac{d\vec{k}}{(2\pi)^3}\, \phi_0^2(k) \int \frac{d\vec{u}}{2\,\pi}\, \delta(1-\vec{u}^{\,2})\, \frac{M^2}{(u\cdot P)^2}\,, \tag{6}$$

where $\phi_0(k)$ represents a solution of a mass operator. Another aspect of the present point form concerns the velocity operator, $\vec{V}$, entering the construction of the Poincaré algebra, and the corresponding $\vec{P}$. Their expressions, which differ from earlier ones, read:

$$\vec{V} = \frac{1}{M}\Big(\vec{p} + \vec{u}\,\frac{4\,m\,\tilde{V}}{2\,u\cdot P}\Big),\ \vec{P} = \vec{p} + \vec{u}\,\frac{4\,m\,\tilde{V}}{2\,u\cdot P} \quad \Big(\text{earlier “P.F.”} : \vec{V} = \frac{\vec{p}}{2\,e_k},\ \vec{P} = \frac{M}{2\,e_k}\,\vec{p}\Big)\,. \tag{7}$$

Despite unusual features, the present point form can be consistently developed. It evidences similarities with the instant and front forms in that the hypersurface it is formulated on is independent of the system under consideration, which is at the origin of the above constraints. These ones are absent in the earlier “point form” where the kinematic character of boosts is trivial, the operation affecting both the system and the hyperplane used for its description, their respective velocity and orientation being related.

3. SOLVED AND UNSOLVED PROBLEMS, DISCUSSION, OUTLOOK

For a part, the present work was motivated by the drawbacks that an earlier point form evidences for the form factors of strongly-bound systems calculated in the single-particle current approximation [3, 4, 12]. Some results are presented in Fig. 1 for the pion charge form factor. At high Q^2, the new point form (D.P.F.) shows a Q^{-4} behavior, like the instant- and front-form results, while the earlier point form (“P.F.”) is providing a Q^{-8} one. The change in the power law is largely due to the form of the velocity operator, Eq. (7), which, containing some dependence on $\hat{u}$, makes less difficult to match the initial- and final-state momenta with those of the struck constituent. At low Q^2, the new point form does better than the earlier one but the improvement is not impressive. More important however, the bad behavior is shared by results obtained in the instant and front forms with parallel kinematics (I.F.+F.F.(parallel)) [12]. All of them evidence a charge squared radius scaling like the inverse of the squared mass of the system. In comparison, the standard instant and front forms (I.F. (Breit frame) and F.F. (perp.) in Fig. 1) do well.

Lorentz invariance of form factors is often considered as an important criterion for validating an approach. With this respect, the point form is to be favored as it fulfills this property. It however recently appeared that the approaches that give bad results above are strongly violating another important symmetry: Poincaré space-time translation invariance [12]. Contrary to Lorentz invariance, this symmetry cannot be checked by looking at form factors in a different frame. Instead, one could check relations such as:

$$\langle\Big[P^{\mu}\ ,\ J^{\nu}(x)\Big]\rangle = -i\langle\partial^{\mu}\, J^{\nu}(x)\rangle. \tag{8}$$

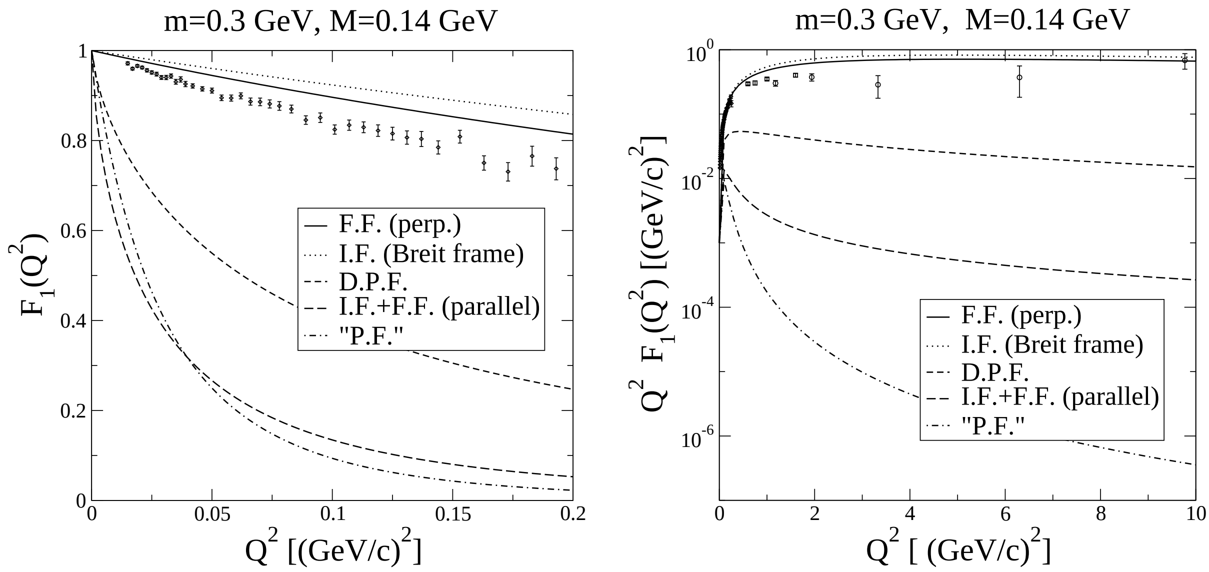

Figure 1. Pion charge form factor in different forms of relativistic quantum mechanics.

This relation [13] cannot be verified exactly at the operator level in RQM approaches with a single-particle current but one can require it is verified, at least, at the matrix-element level. With this respect, what is an advantage for the point-form approach becomes a disadvantage as there is no frame where one can minimize the effect of a violation of the above relation (a factor 2-3 for the nucleon and roughly 6 for the pion). On the contrary, in the instant and front forms, one can consider different frames. It turns out that the instant- and front-form results for a perpendicular kinematics (standard ones) verify the above equality while those for a parallel kinematics do badly, similarly to the point-form case. It thus appears that Poincaré space-time translation invariance could be more important than the Lorentz one and that the intrinsic Lorentz covariance of the point-form approach is not so much an advantage than what could be *a priori* expected.

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